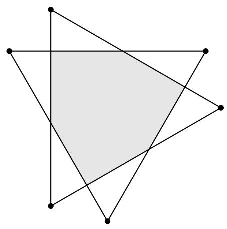
4701. A function f is defined, over the real numbers, such that the following conditions are fulfilled:

① $f(x) = |x|$ for $x \in [-1, 1]$,
② f is periodic with period 2.

Show that $f(x) - \frac{1}{4}x = 0$ has four roots.

4702. Two equilateral triangles, each with area 1, are centred on the same point. At three of the points of intersection, the edges intersect at right-angles.



Show that the area of the region in common to both triangles is $\sqrt{3} - 1$.

4703. Show that $\displaystyle\int_0^\pi x\cos^2 x\,dx = \frac{\pi^2}{4}$.

4704. A large hollow cube has a small smooth sphere of weight $W$ inside it. The cube is placed with one of its space diagonals vertical, and the sphere rests in equilibrium above the lowest vertex. Find the contact force exerted by the sphere on each of the faces it is in contact with.

4705. An equation is given as

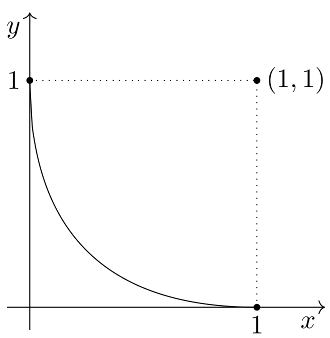
$$\sin(2x + 1) = \cos(2x - 1).$$

Show that the general solution of this equation is

$$x = \frac{(1 + 4n)\pi}{8}, \text{ for } n \in \mathbb{Z}.$$

4706. A curve in the $(x, y)$ plane is defined by

$$\sqrt{x} + \sqrt{y} = 1.$$




Show that the distance between the curve and the point $(1, 1)$ is never less than 1.

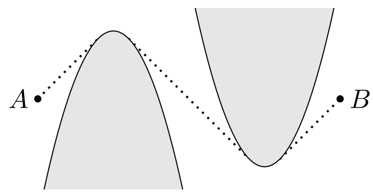
4707. An arithmetic sequence and a geometric sequence, all of whose terms are positive, have the same first term $a$ and the same third term $b$. Prove that the second term in the arithmetic sequence is greater than or equal to the second term in the geometric sequence.

4708. Show, by integration, that

$$\int 41e^{4x}\sin 5x\,dx = e^{4x}(4\sin 5x - 5\cos 5x) + c.$$

4709. The quartic graph $y = 4x^4 + 4x^3 + kx^2 - 2x + 1$, where $k$ is a constant, has two distinct stationary points on the $x$ axis. Find their $x$ coordinates.

4710. A journey through a mountain pass, from $A$ to $B$, is modelled with the following simplified map, showing a **plan** view. In units of kilometres, $A$ is at $(-5, 0)$, $B$ is at $(5, 0)$, and the shaded regions, bounded by the parabolae $y = (x - 1)(x - 4)$ and $y = -(x + 1)(x + 4)$, represent impassable ground.




Show that, taking the shortest possible route, the distance travelled in the central section, between the boundary curves, is $4\sqrt{2}$ kilometres.

4711. In the special theory of relativity, the energy $E$ of a particle moving at speed $v$ is given, in terms of mass $m$ and the speed of light $c$, by

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

This can be approximated, for $v$ significantly less than $c$, with a polynomial

$$E \approx k_0 + k_1v^2 + k_2v^4.$$

Determine the approximation.

4712. The following equation has infinitely many roots:

$$e^x + \cos x = e^x \sin^2 x.$$

Prove that, as $x \to \infty$, these roots occur in pairs $x_a < x_b$, where $x_b - x_a \to 0$.

4713. By first completing the square, show that

$$\int_{-\infty}^{\infty} \frac{3}{9x^2 + 6x + 2}\,dx = \pi.$$

4714. Show that, for $x > 1$,

$$2y - 1 = (x - 1)(y - 1)^2$$
$$\implies y = \frac{\sqrt{x}}{\sqrt{x} \pm 1}.$$

4715. Prove by contradiction an ancient result of Euclid, that there are infinitely many prime numbers.

4716. Quartic functions g and h have the same second derivative. Also, $\text{h}(x) > 0$ for all $x$.

(a) Show that $\text{g}(x) - \text{h}(x)$ is linear in $x$.

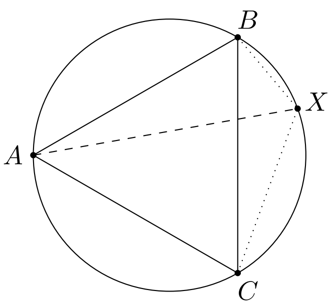
(b) Show that, if $\text{g}(x) - \text{h}(x)$ is constant, then there is at least one stationary value of

$$y = \frac{\text{g}(x)}{\text{h}(x)}.$$

4717. An equilateral triangle $ABC$ is inscribed in a unit circle, with vertices at

$$\begin{aligned} A &: (-1, 0), \\ B &: \left(1/2, \sqrt{3}/2\right), \\ C &: \left(1/2, -\sqrt{3}/2\right). \end{aligned}$$

Point $X$ lies on minor arc $BC$, with coordinates $(\cos\theta, \sin\theta)$, where $\theta \in (-\pi/3, \pi/3)$.




You are given the following half-angle identity:

$$\sqrt{2 + 2\cos\theta} \equiv 2\cos\tfrac{1}{2}\theta.$$

(a) Show that $|AX| = 2\cos\frac{1}{2}\theta$.

(b) Hence, prove that $|AX| = |BX| + |CX|$.

4718. Sketch the following graphs, in which $a$ and $b$ are constants, on the same set of axes, labelling any points of tangency:

① $(x-a)^2 (y-b)^2 = 1$,
② $(x-a)^2 + (y-b)^2 = 2$.

4719. The quadratic approximation to the exponential function, for small $x$, is

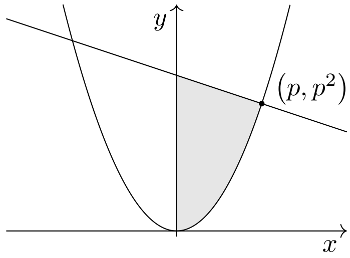
$$e^x \approx 1 + x + \tfrac{1}{2}x^2.$$

Show that, for small $x$,

$$e^x \approx 2 + \sqrt{2}\sin\left(x - \tfrac{\pi}{4}\right).$$

4720. Use integration, with respect to an angle $\theta$, to show that the average value of the quantity $|xy|$ on the unit circle is $1/\pi$.

4721. A normal is drawn to $y = x^2$ at $x = p$, where $p > 0$. Together with the $y$ axis, parabola and normal enclose a region of area $A$.




Show that $A = \frac{2}{3}p^3 + \frac{1}{4}p$.

4722. Point $P : (1/2, 1/2)$ lies on the parametric curve

$$\begin{aligned} x &= \cos 2t, \\ y &= 4\sin^3 t. \end{aligned}$$

The tangent at $P$ meets the curve again at $Q$. Show that the coordinates of $Q$ are $(7/8, -1/16)$.

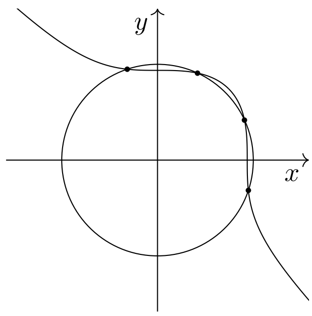
4723. Sketch the inequality

$$y^2 + \sin x\cos x - y\sin x - y\cos x \leq 0.$$

4724. Four forces, whose magnitudes are $1, 2, 3$ and $4$ Newtons, act on an object. The equations of their lines of action are, not necessarily in order, $x = 1, 2, 3$ and $4$. Determine whether it is possible for the object to remain in equilibrium.

4725. Show that $\displaystyle\int_0^{\frac{\pi}{2}} \sin^3 x \, dx = \tfrac{2}{3}$.

4726. The diagram shows the curve $x^3 + y^3 = 26$ and a circle centred on the origin with radius $\sqrt{10}$:




Find the coordinates of the points of intersection, giving non-exact answers to 3sf.

4727. A quintic function f is defined as

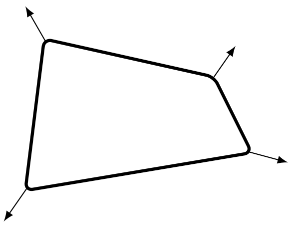
$$\text{f}(x) = 1 + x + \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 + \tfrac{1}{6}x^4 + \tfrac{1}{20}x^5.$$

Show that f is increasing on $\mathbb{R}$.

4728. Show that there are no constants $A, B \in \mathbb{R}$ for which the following identity holds:

$$\frac{x^2 + 2x + 3}{x^3 + x} \equiv \frac{A}{x} + \frac{B}{x^2 + 1}.$$

4729. A loop of smooth, light, inextensible rope is pulled taut by four coplanar forces.



Prove that the intersections of the lines of action of adjacent forces form a cyclic quadrilateral.

4730. A graph is defined, for $t \in [0, 2\pi)$, by

$$x = \cos t,$$
$$y = \sin 2t.$$

(a) Find the Cartesian equation of the graph.

(b) Show that the graph has exactly one point of self-intersection.

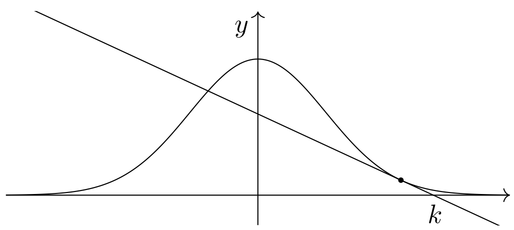
4731. After a sudden ecological change, a population of beetles is adjusting to new conditions. Population size $P$, measured above a baseline at the historical size, is modelled with the DE

$$\frac{d^2P}{dt^2} + 4\frac{dP}{dt} + 13P = 0.$$

(a) By differentiating and substituting, show that $P = e^{-2x} \sin kx$, where $k > 0$ is a constant to be determined, satisfies the DE.

(b) Sketch this curve for $t \in [0, \infty)$, and describe the predicted behaviour of the population.

4732. A straight line passes through $(k, 0)$ and is tangent to the curve $y = e^{-\frac{1}{2}x^2}$.




Show that $|k| \geq 2$.

4733. A hollow sphere of radius $R$ has a cylinder inside it. Prove that the volume of the cylinder satisfies

$$V \leq \frac{4\pi R^3}{3\sqrt{3}}.$$

4734. A curve has equation

$$y = \frac{\sqrt{x^2 - 1} - 1}{x}.$$

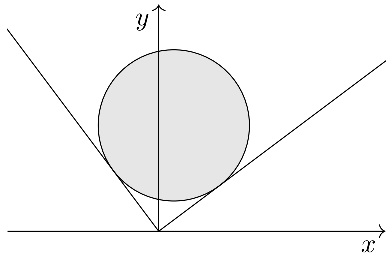
Find the exact coordinates of the points on the curve where the gradient is $\frac{3}{10}$.

4735. The function h is a positive, monic quartic, whose four roots are in arithmetic progression. The graph $y = \mathrm{h}(x)$ has the $y$ axis as a line of symmetry. Show that, for some constant $k$,

$$\mathrm{h}(x) = x^4 - 10k^2x^2 + 9k^4.$$

4736. Show that $2 \arctan \frac{1}{2} = \arcsin \frac{4}{5}$.

4737. A tree trunk of weight $W$ rests on an adjustable lumberyard workbench. In cross-section, this is modelled as a circle of unit radius resting on the lines $y = -m_1x$ and $y = m_2x$, where $m_1, m_2 > 0$.




Neglecting friction, show that

$$R_1 = \frac{m_2\sqrt{m_1^2 + 1}}{m_1 + m_2}W,$$

and give the corresponding result for $R_2$.

4738. A differential equation is given as

$$\frac{dy}{dx} = \sec^2 3y\sqrt{x^4 + 2x^2}.$$

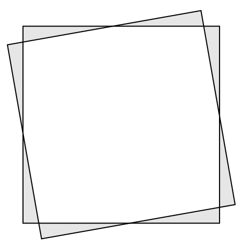
Show that the general solution can be written as

$$6y + \sin 6y = 4(x^2 + 2)^{\frac{2}{3}} + c.$$

4739. Solve the following equation, for $\theta \in [0, 2\pi)$:

$$3 \sin 2\theta - 8 = 10 \cos\left(\theta + \arctan \tfrac{3}{4}\right).$$

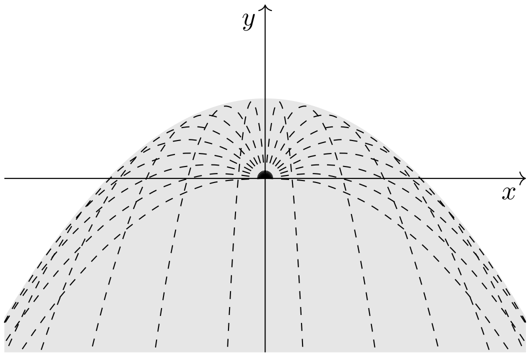
4740. A square $S_1$ has sides of length 1. It is rotated by a small angle $\theta$, measured in radians, around its centre to form a second square $S_2$. This generates a set of eight congruent triangles, shaded below.



Show that the total area of the congruent triangles is approximately $\theta(1 - \theta)$.

4741. A fair six-sided die is rolled four times, and each score turns out to be higher than the previous one. Find the probability that the last score was a six.

4742. In a vertical $(x, y)$ plane, particles are projected from the origin with fixed speed $u$. The angle of projection is variable. The set of $(x, y)$ points attainable by the particles is shaded below. Its boundary is known as the *parabola of safety*.




Find the equation of the parabola of safety.

4743. Consider the following equation, in which $n, k \in \mathbb{N}$:

$$^{n}\mathrm{C}_{k} + {}^{n}\mathrm{C}_{k+1} = {}^{n}\mathrm{C}_{k+2}.$$

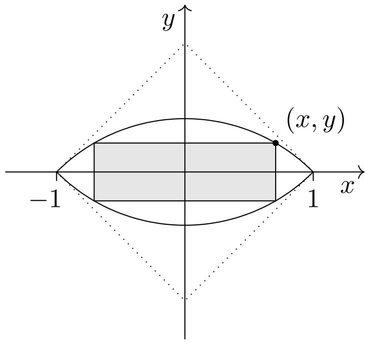
You are given that $k$ is a single-digit number.

(a) Show that $n = \dfrac{3k + 3 \pm \sqrt{5k^2 + 14k + 9}}{2}$.

(b) Hence, find $n$ and $k$.

4744. Show that $\displaystyle\int_0^1 \frac{1}{\sqrt{x} + 1}\, dx = 2 - \ln 4$.

4745. On the axes below, two quarter-circles meet at right angles. A rectangle is drawn with all four vertices on the quarter-circles. Its vertex in the positive quadrant has coordinates $(x, y)$.




Find the exact value of $x$ for which the area is a maximum. Give your answer in the form

$$x = \sqrt{\tfrac{a + b\sqrt{c}}{d}}.$$

4746. Prove from first principles that, if $y = x^{\frac{1}{3}}$, then

$$\frac{dy}{dx} = \tfrac{1}{3}x^{-\frac{2}{3}}.$$

4747. A *fourth-order* differential equation is given by

$$\frac{d^4y}{dx^4} - 2\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0.$$

Verify that $y = xe^x$ is a solution.

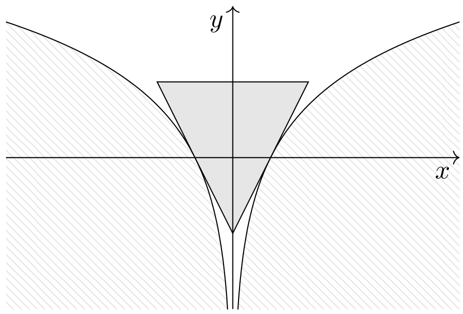
4748. A curve is defined implicitly by

$$x^2y + xy^2 = 2.$$

Three points $P, Q, R$ are labelled on the curve. You are given that the tangent at $P$ is parallel to $x$, the tangent at $Q$ is parallel to $y$, and the tangent at $R$ has gradient $-1$. Show that the triangle formed by the three tangents has area 18.

4749. Solve the inequality $(5 - x)(5 - |x|) > 9$.

4750. A smooth funnel is shown below in cross-section. The curved surface of the funnel is described by rotating part of the graph $y = \ln(x^2)$ around the vertical $y$ axis. A conical plug of mass $m$ is placed in the funnel. Its circle of contact with the inner surface of the funnel is represented in the cross-section by the points $(\pm 1, 0)$, in units of cm.




Determine the force exerted, per centimetre of the circle of contact, by the conical plug on the funnel.

4751. A *random walk* on a two-dimensional grid involves a sequence of steps, starting at the origin, each of which, with equal probability, is a move of either $\pm\mathbf{i}$ or $\pm\mathbf{j}$. Five steps are made.

(a) Explain why the probability of ending up at the point $(1, 1)$ is zero.

(b) Find the probability of ending up at $(3, 0)$.

4752. Solve, for $0 \leq \psi < 2\pi$, the equation

$$4 \tan 2\psi + 3 \cot \psi \sec^2 \psi = 0.$$

4753. The parabola $y = ax^2 + bx + c$ has $x$ intercepts at $x = p, q$. Determine, with coefficients in terms of $a, b, c$, the equation of the monic quartic which has a stationary point at the origin and points of inflection at $x = p, q$.

4754. An iteration $x_{n+1} = \mathrm{f}(x_n)$ has starting value $x_0$, where $x_0 > 0$. The function f is defined as

$$\mathrm{f}(x) = 2x^4 - 2x^3 + x^2 + x.$$

Show that

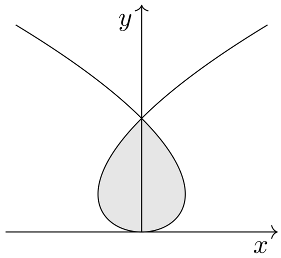
(a) $\mathrm{f}'(x) > 1$ for $x > 0$,

(b) $x_{n+2} + x_n > 2x_{n+1}$ for all $n \in \mathbb{N}$.

4755. A graph is given, for some constant $k \in \mathbb{N}$ and a parameter $t \in \mathbb{R}$, by

$$x = t^{2k+1} - t^{2k-1},$$
$$y = t^{2k}.$$

The graph encloses a region of area $A$:




Show $A = \dfrac{8k}{16k^2 - 1}$.

4756. In an experiment, three events $A$, $B$ and $C$ have probabilities as follows:

$$\mathbb{P}(A) = \tfrac{1}{2}, \quad \mathbb{P}(B) = \tfrac{1}{2}, \quad \mathbb{P}(C) = \tfrac{3}{4}.$$

Find all possible values for $\mathbb{P}(A \cap B \cap C)$, giving your answer in set notation.

4757. Sketch the region of the $(x, y)$ plane which satisfies

$$\frac{y - x^2 + 1}{y + x^2 - 1} < 1.$$

4758. A particle oscillates with horizontal and vertical positions $x$ and $y$, in units of metres, given as:

$$x = 3 \sin kt, \quad y = 19.6.$$

This continues until, at a randomly chosen time, all forces but gravity are removed, and the particle becomes a projectile. Find $k$ such that the set of attainable points on the $x$ axis is $x \in [5, 5]$.

4759. A function of $x$ and $y$ is defined as

$$\mathrm{f}(x, y) = 3x^2 + 2xy + 3y^2.$$

(a) Write $\mathrm{f}(x, y)$ as $a(x + y)^2 + b(x - y)^2$, where $a$ and $b$ are to be determined.

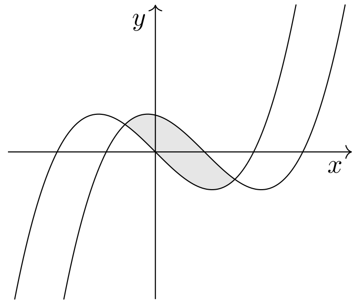
(b) Hence, show that $\mathrm{f}(x, y) = 1$ is an ellipse.

4760. The equations of two cubics are given as follows, with $k$ as a constant:

$$y = x^3 - x,$$
$$y = (x - k)^3 - x + k.$$

(a) Find all values of $k$ for which the cubics have at least one point of intersection.

(b) The constant $k$ is now set as $k = \frac{1}{2}$. The graphs of the cubics are as shown.




Find, to 4sf, the radius of the largest circle which can be inscribed in the shaded region.

4761. Brahmagupta studied triangles with side lengths $a, b, c$ generated by

$$a = n\big(m^2 + k^2\big),$$
$$b = m\big(n^2 + k^2\big),$$
$$c = \big(m + n\big)\big(mn - k^2\big).$$

Prove that the area of such a triangle is

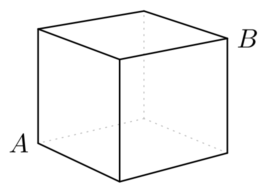
$$A = mnk\big(m + n\big)\big(mn - k^2\big).$$

You may wish to use Heron's formula for the area of a triangle, in terms of the semiperimeter $s$:

$$A = \sqrt{s(s - a)(s - b)(s - c)}.$$

4762. Show that the region defined by $x^3y - xy^3 \geq 1$ is tangent to the circle $x^2 + y^2 = 2$ at four points.

4763. An ant is walking on a wire-frame cube.




Show that, if the ant never revisits a vertex, then there are eighteen routes from $A$ to $B$.

4764. A graph has equation

$$y = \frac{1}{x} - \frac{1}{x^3}.$$

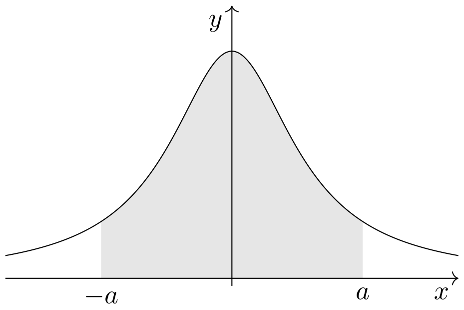
Find the set of possible values of $k$ such that the line $y = k$ intersects the curve exactly three times.

4765. (a) Find $a, b \in \mathbb{Q}^+$ such that

$$\frac{\sqrt{3-\sqrt{5}}}{2\sqrt{2}} = a\sqrt{5} - b.$$

(b) You are given that $\cos 36° = \frac{1}{4}\big(1+\sqrt{5}\big)$. Show that $\sin 18° = \frac{1}{4}\big(\sqrt{5}-1\big)$.

4766. The graph below shows $y = \dfrac{3}{1+x^2}$:




The shaded region has area $2\pi$. Find $a$.

4767. A polynomial function h has odd degree. There are exactly two $x$ values at which $\mathrm{h}''(x) = 0$. Prove that $y = \mathrm{h}(x)$ has exactly one point of inflection.

4768. Prove that the cube root of 5 is irrational.

4769. Two $(x, y)$ graphs are defined by

$$\begin{aligned} &(x-y)^3 - k(x-y) = 0, \\ &x^2 + y^2 = 4. \end{aligned}$$

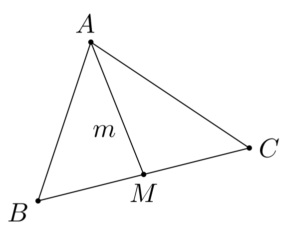
Find the value of the constant $k$ for which the graphs have exactly four points of intersection.

4770. A cubic equation is defined, for $a, b \in \mathbb{R} \setminus \{0\}$, as

$$ax^3 + ax^2 + ax + b = 0.$$

Given that $b$ is a root, show that $-\frac{4}{3} \le a \le 0$.

4771. In $\triangle ABC$, with side lengths $(a, b, c)$ defined in the usual way, the median joining $A$ to the midpoint of $BC$ has length $m$.




Prove that $m = \sqrt{\dfrac{b^2+c^2}{2} - \dfrac{a^2}{4}}$.

4772. It is possible to cut a cube in half, such that the cross-section is a regular hexagon. Determine the ratio of side lengths of the cube and the hexagon.

4773. A function G and its derivative g are defined over the real numbers. The graph of $y = \mathrm{G}(x)$ passes through the points $(0, 0)$, $(a, b)$, and $\big(a^2, b^2\big)$.

(a) Show that $\displaystyle\int_0^a \mathrm{g}(x)\, dx = b$.

(b) In terms of $b$, find $\displaystyle\int_0^a x\, \mathrm{g}\big(x^2\big)\, dx$.

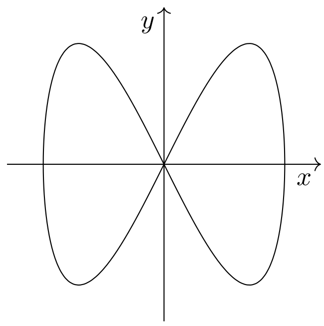
4774. In this question, $k$ is a non-zero constant.

Find the number of real roots of

$$\big(x^2 + x + k^2 + 1\big)\big(x^3 - x - k^2 - 1\big)\big(x^6 + k^4\big) = 0.$$

4775. Let $A_n$ be the area of the region of the $(x, y)$ plane defined by the inequality $x^{2n} + y^{2n} \le 1$, for $n \in \mathbb{N}$. Find the value of $\lim\limits_{n\to\infty} A_n$.

4776. The Lissajous curve shown below is defined, for $t \in [0, 2\pi)$, by the equations $x = \cos t$, $y = \sin 2t$:




(a) Find the Cartesian equation of the curve.

(b) Show that the total area enclosed is $\frac{8}{3}$.

4777. An equation is given as $\sin(\ln x) - x = 0$. Prove that this equation has infinitely many roots.

4778. Two functions f and g are defined to be positive, monic quadratics. Each of the equations $\mathrm{f}(x) = 0$ and $\mathrm{g}(x) = 0$ has two distinct, positive roots, at $x = a, b$ and $x = b, c$ respectively. The roots $a, b, c$ are in geometric progression, with common ratio $r > 1$. Prove that the equation $\mathrm{f}(x) + \mathrm{g}(x) = 0$ has exactly two roots.

4779. Show that no $x$ values satisfy the equation

$$|x-1| + |x+1| = 1 - |x|.$$

4780. A projectile is launched from an origin, with initial speed $u$. The angle $\theta$ above the horizontal is set so that the projectile hits point $(p, q)$. Prove that

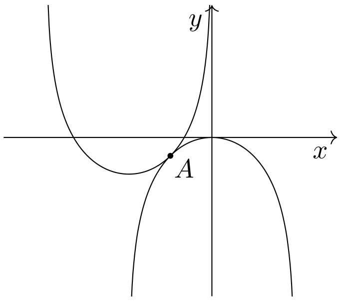
$$\tan\theta = \frac{u^2 \pm \sqrt{u^4 - g(2qu^2 + gp^2)}}{gp}.$$

4781. Sketch the curve $x^2y + y^2 = 1$.

4782. A regular tetrahedron is placed with a face flat to horizontal ground. Prove that the gradients of the other faces are twice the gradients of their edges.

4783. The curve shown below is given implicitly by

$$\sin 2x - 2e^y \sin x + e^{-y} \cos x - 1 = 0.$$




Find the exact coordinates of point $A$, where the two branches of the graph are tangent.

4784. Differentiate $y = x^{\frac{1}{4}}$ from first principles.

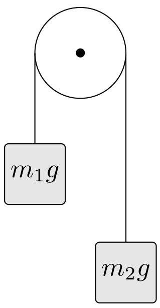
4785. The position of a swinging pendulum, relative to an origin $O$, is modelled as $x = e^{-t} \sin 2t$. This function satisfies a DE of the following form, in which $a$ and $b$ are constants:

$$\frac{d^2x}{dt^2} + a\frac{dx}{dt} + bx = 0,$$

where $a, b \in \mathbb{R}$. Determine $a$ and $b$.

4786. Factorise $x^5 - 6x^4 + 6x^3 - 36x^2 + 9x - 54$ fully.

4787. Two blocks, with masses $m_1 > m_2$, are connected by a light, inextensible string passed over a rough pulley. Friction causes the tensions on either side of the pulley to differ by a maximum of $\mu R$ N, where $R$ is the total downward force exerted on the pulley by the string.




Show that, if this rough system moves, then its acceleration is the same as the equivalent smooth system with masses $(1 - \mu)m_1$ and $(1 + \mu)m_2$.

4788. Solve $x^{24} - 3x^{15} + 3x^6 = x^{-3}$.

4789. Show that, if $\sin^2(x + y) + \cos^2(x - y) = 1$ holds, then either $x$ or $y$ is a multiple of $\frac{1}{2}\pi$.

4790. In this question, do not use a calculator.

You are given that $y = 36x^4 + 12x^3 - 11x^2 - 2x + 1$ has two local minima on the $x$ axis. Determine their $x$ coordinates.

4791. Using a substitution of the form $u = f(x, y)$, find the general solution of the differential equation

$$\frac{dy}{dx} = x + y.$$

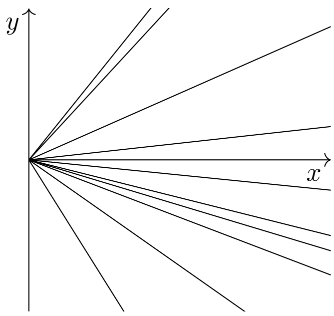
4792. Show that the parametric equations $x = 2t - t^2$, $y = 2t + t^2$ define a parabola, and sketch it.

4793. A function is given, over $\mathbb{R} \setminus \{0, -2\}$, by

$$f(x) = \frac{x^2 + 2x + 1}{x^2 + 2x}.$$

Determine the range of this function, giving your answer in interval set notation.

4794. In the half-plane $x \geq 0$, lines through the origin are generated randomly. Their equations are $y = Mx$, where the gradient $M$ has a normal distribution $M \sim N(0, 1)$. A random sample of ten such lines is shown below:




A line is generated at random.

(a) Determine the probability that the line passes between the points $(1, 1)$ and $(1, 2)$.

(b) Given that the line has a positive gradient, find the probability that it is steeper than $y = x$.

4795. Prove, using integration by substitution, that

$$\int \frac{1}{x\sqrt{x^2 - 1}}\, dx = \text{arcsec}\, x + c.$$

4796. Factorise $-x^3y - xy^2 - xy + y + 1 + x^2$.

4797. It is given that the following equation has exactly one real root $x \in [0, 2\pi)$:

$$\cos^2 x - 2\cos x + k = 0.$$

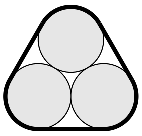
Determine all possible values of the constant $k$.

4798. A positive cubic graph $y = \mathrm{f}(x)$ has distinct SPs at $x = p$ and $x = q$. The function g is defined, in terms of f and a constant $k$, as

$$\mathrm{g}(x) = \mathrm{f}(x + k) - \mathrm{f}(x - k).$$

Show that $\mathrm{g}(x)$ is minimised at $\frac{1}{2}(p + q)$.

4799. On a polar expedition, three cylindrical ice cores each of radius $r$ and weight $W$ are bound together with a loop of light rope, as shown below in cross-section. All contacts are modelled as smooth.



The cores are placed on a trestle, which exerts a symmetrical reaction force upwards on the two lower cores, but doesn't touch the rope. Show that, if equilibrium is to be maintained, the minimum possible value of the tension in the rope is

$$T = \frac{\sqrt{3}}{6}W.$$

4800. Show that, for constants $a, b, k \neq 0$,

$$\int \frac{a + e^{kx}}{b + e^{kx}}\, dx = \frac{(b - a)\ln|b + e^{kx}| + akx}{bk} + c.$$

——— End of 48th Hundred ———